Gauss law: relates electric fields to source: charge

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

electric field lines start/stop on charges

Magnets have no magnetic charges so

$$\oint \vec{B} \cdot d\vec{A} = 0$$

magnetic field lines never end on anything

Source of magnetic fields:

currents can generate $\vec{B}$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

relates changing $\vec{E}$ to $\vec{B}$

Faraday's law relates $\vec{E}$ to changing $\vec{B}$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

if there are magnetic charges, then currents I_B would also generate $\vec{E}$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} + (\ ) I_B$$

Faraday's law:

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

if surface is constant:

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{A}$$

a changing $\vec{B}$ field generates $\vec{E}$

Ampere's law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{inside} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

if there are no currents (I inside loop $=0$) and area of loop is constant:

$$\oint \vec{B}\cdot d\vec{l} = \varepsilon_0\mu_0 \int \frac{d\vec{E}}{dt}\cdot d\vec{A}$$

so a changing $\vec{E}$ field generates $\vec{B}$

These 2 equations tell us a great deal:

① $\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$

② $\oint \vec{B}\cdot d\vec{l} = \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{s}$

Eq ① relates $\vec{E}$ to changing $\vec{B}$

Eq ② " $\vec{B}$ " " $\vec{E}$

- person ① holds charge Q

initial: $\vec{B}=0$, $\vec{E}$ = constant

- ① wiggles charge: this cause changing E that propagates to person ②

$\frac{d\vec{E}}{dt} \neq 0$ ∴ this causes $\vec{B} \neq 0$

- $\vec{B}$ goes from $B=0$ to nonzero value so $\frac{dB}{dt} \neq 0$ ∴ this cause $\vec{E} \neq 0$

the 2 fields induce a propagating wave

What is $\varepsilon_0 \mu_0$?

$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ $\quad$ $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$

$$\varepsilon_0\mu_0 = 8.85 \times 10^{-12} \cdot 4\pi \times 10^{-7} \frac{C^2}{Nm^2} \frac{N}{(C/s)^2}$$

$$= 1.112 \times 10^{-17} \frac{s^2}{m^2} \Big\} \text{ units of } \frac{1}{\text{velocity}^2}$$

$$= \frac{1}{(3 \times 10^8 \, m/s)^2}$$

so $\frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3 \times 10^8 \, m/s$ units of velocity

this is a fundamental unit of nature!

after some calculation can show that

$\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ = speed of propagation of EM waves

= speed of light c

Conundrum: vel of EM waves is constant? does not depend on reference frame?

⇒ Discuss briefly why this is not intuitive and will be important for ch 37 relativity

This describes system for radiating energy away from moving charges

$\Rightarrow$ EM radiation!

Discuss EM spectrum (slide 3 & 4)

Mathematics of Maxwell's equations

in integral form, full equations

Gauss' Law: $\oint \vec{E}\cdot d\vec{S} = Q_{inside}/\varepsilon_0$

charge inside surface S

No magnetic charges: $\oint \vec{B}\cdot d\vec{S} = 0$

Faraday's Law: $$\underbrace{\oint \vec{E}\cdot d\vec{\ell}}_{\text{EMF around loop enclosing } S} = -\frac{d}{dt}\underbrace{\int \vec{B}\cdot d\vec{S}}_{\Phi_B \text{ flux surface } S}$$

Ampere's Law: $$\underbrace{\oint \vec{B}\cdot d\vec{\ell}}_{\vec{B} \text{ around loop } \ell \text{ enclosing surface } S} = \mu_0\Big(\underbrace{I_{inside}}_{\text{current inside loop}} + \varepsilon_0 \frac{d}{dt}\underbrace{\int \vec{E}\cdot d\vec{S}}_{\Phi_E \text{ flux surface } S}\Big)$$

For EM waves, fields are in a "vacuum"
(no source charges where fields propagate)

$\Rightarrow$ this gives the 4 equations

$\oint \vec{E} \cdot d\vec{s} = 0$	$\int \vec{B} \cdot d\vec{s} = 0$
$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{s}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$

Now for some vector calculus

Define "gradient" operator

$$\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$\vec{\nabla}$ is a vector that operates on a vector field, e.g.

let $\vec{F}(x,y,z) = F_x(x,y,z)\hat{i} + F_y(x,y,z)\hat{j} + F_z(x,y,z)\hat{k}$

then can define "divergence of $\vec{F}$":

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

can also define "curl of $\vec{F}$"

$$\vec{\nabla} \times \vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right)$$

this gives lots of terms, from $\hat{i}\times\hat{i}$ & $\hat{i}\times\hat{j}$ & $\hat{i}\times\hat{k}$ etc

⇒ but $\hat{i}\times\hat{i} = 0$ (cross product of parallel vectors)
so only 6 terms remain

⇒ also $\hat{i}\times\hat{j} = \hat{k}$ & $\hat{j}\times\hat{k} = \hat{i}$ & $\hat{k}\times\hat{i} = \hat{j}$

and $\hat{j}\times\hat{i} = -\hat{i}\times\hat{j}$ etc.

after the terms are all collected we get

$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$$

it turns out we won't have to actually use the right hand side of the above equation!

Next we need 2 theorems from vector calculus

Stokes' Thm: $\oint \vec{F}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$

Divergence Thm: $\oint \vec{F}\cdot d\vec{S} = \int (\vec{\nabla}\cdot\vec{F})\, dV$

- for Stokes, $d\vec{l}$ is the closed curve around the surface $d\vec{S}$
- for Divergence, $d\vec{S}$ is the surface around volume dV

Now take the 2 "gauss" equations:

$$\oint \vec{E}\cdot d\vec{S} \to \int (\vec{\nabla}\cdot\vec{E})\,dV = 0 \quad \text{in vacuum}$$

so if the divergence of $\vec{E}$ integrated over a volume is zero then

$$\boxed{\vec{\nabla}\cdot\vec{E} = 0}$$

$\int \vec{B}\cdot d\vec{S} = 0$ gives a similar $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$

next take Faraday's law:

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$

and use Stokes theorem on $\oint \vec{E}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{E})\cdot dS$

to get:

$$\int (\vec{\nabla}\times\vec{E})\cdot d\vec{S} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{S}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

next take Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{s}$$

and use Stokes' on $\oint \vec{B} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{B}) \cdot d\vec{s}$

$$\Rightarrow \quad \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}}$$

These are Maxwell's equations in differential form w/ no sources (in "vacuum")

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \qquad \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

notice that the 2 equations with cross products.
$\Rightarrow$ they imply that $\vec{E}$ and $\vec{B}$ are <u>perpendicular</u>

$\Rightarrow$ so however $\vec{E}$ and $\vec{B}$ move together thru empty space, they will be $\perp$ to each other to see how the fields propagate in empty space we need to combine to get an equation for $\vec{E}$ and for $\vec{B}$ separately

$\Rightarrow$ do this use the following horrible identity:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$$

start w/ $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$ and take $\vec{\nabla} \times$ on both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}\underbrace{(\vec{\nabla} \cdot \vec{E})}_{\text{this is } \emptyset \text{ by Maxwell!}} - \nabla^2 \vec{E}$$

$$\vec{\nabla} \times \left(-\frac{d\vec{B}}{dt}\right) = -\vec{\nabla} \times \frac{d\vec{B}}{dt} = -\frac{d}{dt}(\vec{\nabla} \times \vec{B})$$

then substitute $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$:

$$-\frac{d}{dt}(\vec{\nabla} \times \vec{B}) = -\frac{d}{dt}\left(\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}\right) = -\mu_0 \varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

put both together:

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{d^2\vec{E}}{dt^2}$$

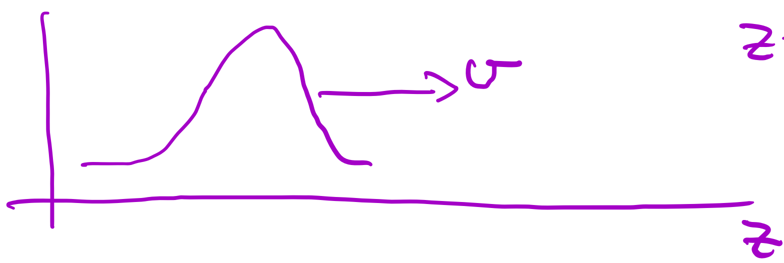
rewrite: $\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{d^2\vec{E}}{dt^2} = 0$

this is the wave equation for $\vec{E}$ in a vacuum!

EM waves – must be a wave equation

what does the wave equation look like?

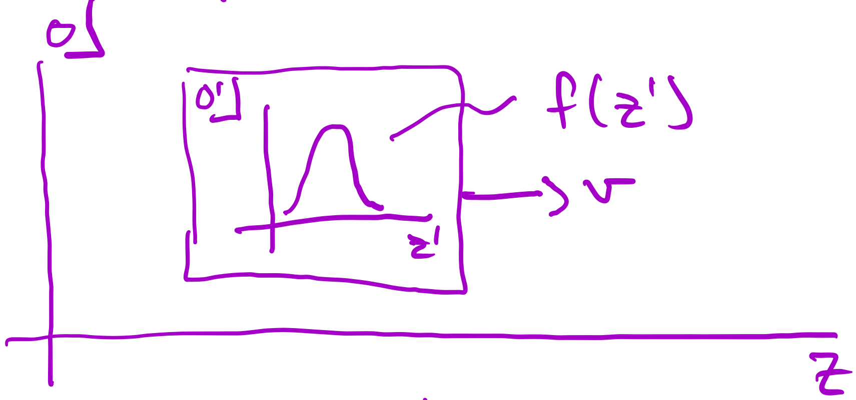
→ imagine any pulse that is propagating along z-axis



in rest frame of pulse it has a shape

⇒ If you run alongside it you will see stationary pulse shape

call that frame O' frame

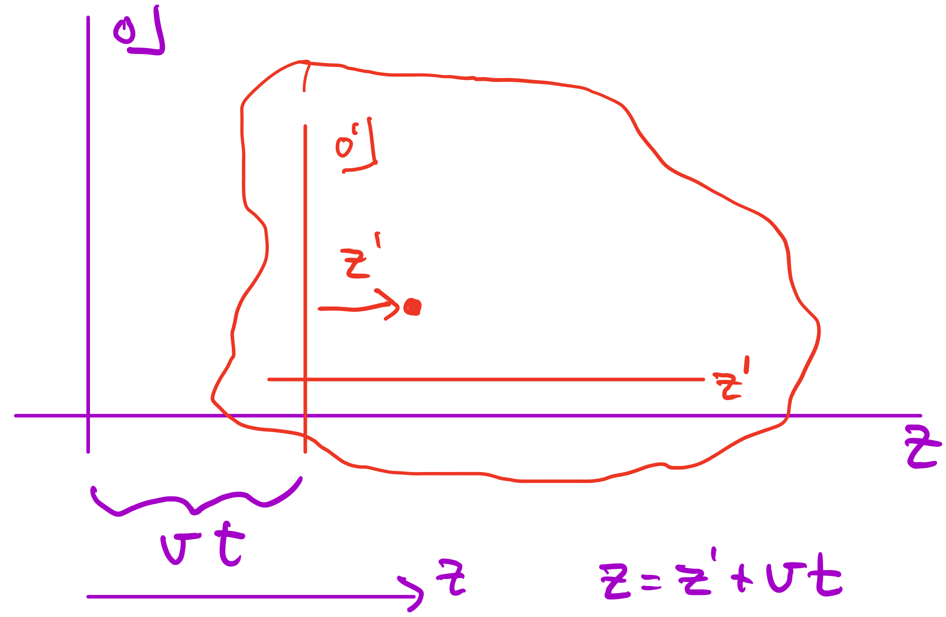


O' is moving w/ velocity v

the coordinate z is related to z' by

$$z = z' + vt$$

to see this ....



$z = z' + vt$

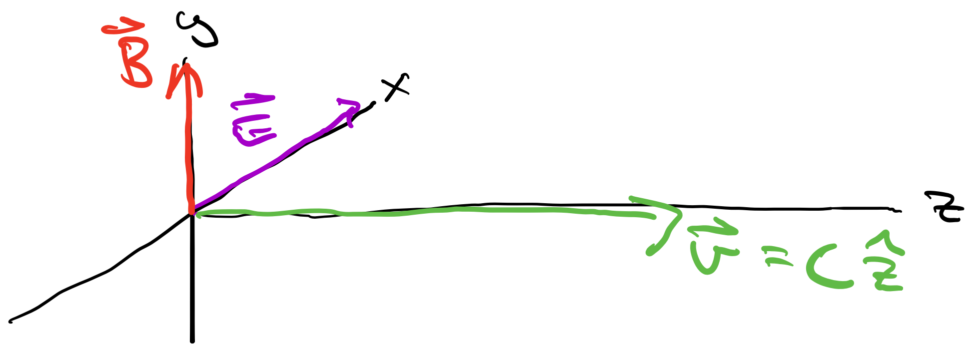
pulse shape in O' is $f(z')$

so pulse shape in O is $f(z - vt)$

this describes a pulse moving along $+z$ dir

EM waves

very far from source, looks like a sine wave



each field oscillates along an axis $\perp$ z dir

$\vec{E}$ along x dir

$\vec{B}$ along y dir

so easy to write down wave function in the wave frame of reference

$$E = E_0 \cos\left(2\pi \cdot \frac{d(t)}{\lambda}\right)$$

λ = dist along z for 1 full period oscillation

and $d(t) = z - vt$

for EM radiation $v = c$

for any wave, λ = dist for 1 osc

T = period of 1 osc

so $\frac{\lambda}{T} = v$ of wave: $v = \frac{\lambda}{T} = \lambda f$

for EM radiation $v = c = \lambda f$

$$2\pi \frac{d(t)}{\lambda} = 2\pi \frac{(z - ct)}{\lambda} = \frac{2\pi}{\lambda} \cdot z - \frac{2\pi(\lambda f)t}{\lambda}$$

$$= \underbrace{\frac{2\pi}{\lambda}}_{k} z - \underbrace{2\pi f}_{\omega} t$$

$$E = E_0 \cos(kz - \omega t)$$

$$B = B_0 \cos(kz - \omega t)$$

for wave along $-z$ direction, $z \to -z$

this gives $E = E_0 \cos(-kz - \omega t) = E_0 \cos(kz + \omega t)$

so in general $E = E_0 \cos(kz \pm \omega t)$

+ for motion along $-z$ dir

− " " " $+z$ "

check using wave equation:

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2}$$

since $\vec{E}$ is only a function of z then

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial z^2} = E_0 \hat{i} \frac{\partial^2}{\partial z^2} \cos(kz - \omega t)$$

$$= E_0 \hat{i} \frac{\partial}{\partial z} \left[-k \sin(kz - \omega t) \right]$$

$$= E_0 \hat{i} (-k^2) \cos(kz - \omega t)$$

$$= -k^2 E_0 \hat{i} \cos(kz - \omega t)$$

$$= -k^2 \vec{E}$$

note: we guessed that $\vec{E}$ pts along x-axis as it moves along z-axis

so $\nabla^2 \vec{E} = -k^2 \vec{E}$

next: $\frac{d^2 \vec{E}}{dt^2} = \frac{d}{dt} E_0 \hat{i} \cos(kx - \omega t) = -\omega^2 \vec{E}$ due to how we take derivatives so the wave equation: $\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{d^2\vec{E}}{dt^2} = 0$

becomes $\quad -k^2 \vec{E} - \mu_0 \varepsilon_0 (-\omega^2 \vec{E}) = 0$

or $\quad (k^2 - \mu_0 \varepsilon_0 \omega^2) \vec{E} = 0$

so this is a solution if $k^2 - \mu_0 \varepsilon_0 \omega^2 = 0$

$\Rightarrow \mu_0 \varepsilon_0 = \frac{1}{c^2}$ "speed of light"

$k = 2\pi/\lambda$ and $\omega = 2\pi/T$

so this becomes

$$\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{2\pi}{T}\right)^2 \frac{1}{c^2} = 0$$

$$\frac{c^2}{\lambda^2} - \frac{1}{T^2} = 0$$

$$c = \lambda / T$$

time to repeat once

distance wave travels before repeating

this proves that

- $\vec{E} = E_0 \hat{i} \cos(kz - \omega t)$ is a wave
- with velocity $v = c = 3 \times 10^8\,\mathrm{m/s}$

$\Rightarrow$ similarly $\vec{B} = B_0 \hat{j} \cos(kz - \omega t)$ and $\vec{B} \perp \vec{E}$

One more puzzle: what does Maxwell's equations say about the amplitudes E_0 & B_0

Take $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$ and plug in solutions

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(E_0 \hat{i} \cos(kz - wt)\right)$$

$$= \left(\underset{\uparrow}{\hat{i}\frac{\partial}{\partial x}} + \underset{\uparrow}{\hat{j}\frac{\partial}{\partial y}} + \hat{k}\frac{\partial}{\partial z}\right) E_0 \hat{i} \cos(kz - wt)$$

zero since

$E(z)$ on

$$= \underbrace{\hat{k} \times \hat{i}}_{\hat{k}\times\hat{i} = \hat{j}} \underbrace{\frac{\partial}{\partial z} E_0 \cos(kz - wt)}_{-w E_0 \sin(kz - w)} = -E_0 w \hat{j} \sin(kz - wt)$$

next: $\frac{d\vec{B}}{dt}$: $\frac{d\vec{B}}{dt} = \frac{d}{dt} B_0 \hat{j} \cos(kz - wt) = -B_0 w \hat{j} \sin(kz - wt)$

so that equation gives

$$-E_0 k \hat{j} \sin(kz - wt) = -B_0 w \hat{j} \sin(kz - wt)$$

$$E_0 = B_0 \frac{w}{k} = B_0 \frac{\lambda}{T} = B_0 c$$

$$\boxed{E_0 = B_0 c}$$ amplitudes are related

Recap: Maxwell's equations tell us:

Electricity & Magnetism are unified into

Electromagnetism (E&M)

- charges generate electric fields
- moving charges generate magnetic fields
- changing $\vec{E}$ field generates $\vec{B}$
- changing $\vec{B}$ generates $\vec{E}$

• these equations describe all E&M phenomena including how EM force propagates in empty space (vacuum)

- in vacuum: $\vec{E} \perp \vec{B}$ and each follows wave equation
- waves propagate at constant

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \equiv c = 3 \times 10^8 \, m/s$$

- if waves propagate along z-direction then

$$\left.\begin{aligned} \vec{E} &= E_0 \hat{\imath} \cos(kz - \omega t) \\ \vec{B} &= B_0 \hat{\jmath} \cos(kz - \omega t) \end{aligned}\right\} \text{these solutions work!}$$

$$E_0 = B_0 c \qquad k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

some problems:

laser has wavelength of 400 nm

E field amplitude is 1000 V/m

B-field amplitude: $E_0 = cB_0$

$$B_0 = \frac{E_0}{c} = \frac{1000\,V/m}{3\times10^8\,m/s}$$

$$= 3.3\times10^{-6}\,T$$

$$= .033\text{ gauss}$$

frequency: $c = \lambda f$

$$f = \frac{c}{\lambda} = \frac{3\times10^8\,m/s}{400\times10^{-9}\,m} = 7.5\times10^{14}\,Hz$$

$$= 750\times10^{12}\,Hz = 750\,THz$$

radio: FM frequencies ~ 100 MHz

wavelength $\lambda = \frac{c}{f} = \frac{3\times10^8\,m/s}{100\times10^6\,/s} = 3m$

AM frequencies ~ 1000 kHz ~ $\frac{1}{100}$ FM

so $\lambda \sim 300m$

FM wavelengths are ~human scale

this is why you can still hear FM radio in short tunnels, under bridges, etc.

⇒ FM signals "fit" into human spaces
AM are too big

a brief aside on speed of light

$$c = 3 \times 10^8 \frac{m}{s}$$

write $3 \times 10^8 \frac{m}{s} = \frac{3\,m}{10^{-8}\,s} = \frac{0.3\,m}{10^{-9}\,s}$

$10^{-9}\,s = 1\,ns$
$0.3\,m \sim 1\,ft$

so $c \sim 1\,ft/ns$ useful to know!

also: $3 \times 10^8 \frac{m}{s} = \frac{0.3\,m}{10^{-9}\,s} = 0.3\,m * 1\,GHz$

$$c = \lambda f = 0.3\,m * 1\,GHz$$

this tells you how to quickly convert from wavelength to freq & back of EM waves

ex: $f = 2\,GHz$ so $\lambda f = .3\,m\text{-}GHz$

$\lambda \cdot 2\,GHz = .3\,m \times 1\,GHz$

$\therefore \lambda = \frac{.3\,m}{2} = .15\,m$

ex: $f = 100\,MHz = \frac{1}{10}\,GHz \Rightarrow \lambda = \frac{.3\,m}{1/10} = 3\,m$

Energy & Intensity in EM waves

capacitor: $U = \frac{1}{2}CV^2$ total energy stored in cap

$C = \frac{\varepsilon_0 A}{d}$ and $V = Ed$ E is electric field across plates

so $U = \frac{1}{2}\frac{\varepsilon_0 A}{d}E^2 d^2 = \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\text{energy/volume}} \cdot \underbrace{Ad}_{\text{volume between plates}}$

$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$ = energy density associated with electric fields

for magnetic fields

solenoid: $U = \frac{1}{2}LI^2$ $B = \mu_0 n I$

$L = \mu_0 n^2 l A$

$U = \frac{1}{2}\mu_0 n^2 l A \cdot \frac{B^2}{\mu_0^2 n^2} = \underbrace{\frac{1}{2}\frac{B^2}{\mu_0}}_{\text{energy/volume}} \underbrace{lA}_{\text{volume in sol}}$

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$ energy density associated with magnetic fields

EM waves have both E & B so must carry energy!

→ that's how radio's work!

Total energy density:

$$u_{EM} = u_B + u_E = \frac{1}{2}\varepsilon E^2 + \frac{B^2}{2\mu_0} \quad \text{but } B = E/c$$

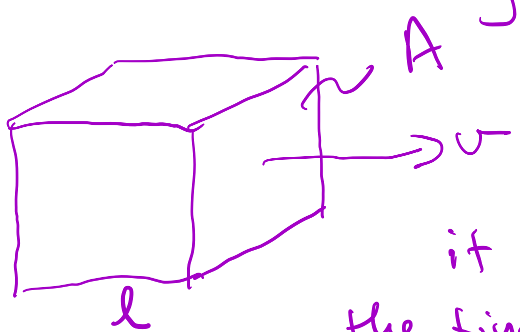
$$= \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{E^2}{2\mu_0 c^2}\right) \quad \text{use } \frac{1}{c^2} = \varepsilon_0 \mu_0$$

$$\boxed{u_{EM} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

not a function of frequency!

u_{EM} units of energy per volume ⇒ $\frac{\text{joules}}{m^3}$

and moves w/ velocity $v = c$



let $E = u_{EM} V$ be amount of energy in box

it moves w/ velocity v so the time for all the energy to move out of the box is given by using $l = vt$

volume $V = lA$

Amount of energy per time is $\frac{E}{t} = \frac{Ev}{l}$

This is total energy per time (power)

Intensity ≡ power per area $= \frac{Ev}{l}/A$

$$= Ev/lA = u_{EM} \cdot v = u_{EM}\, c$$

so intensity $I = u_{EM} c = \varepsilon_0 E^2 c$ in vacuum

using $E = cB$ & $c^2 \varepsilon_0 \mu_0 = 1$ can also write as

$$I = \varepsilon_0 E^2 c = \varepsilon_0 E B c \cdot c = \varepsilon_0 E B c^2 = \frac{EB}{\mu_0}$$

This energy flows along direction ⊥ to both $\vec{E}$ & $\vec{B}$ => $\vec{S}$ = "Poynting vector"

since $E \perp B$ and $\vec{S} \perp$ both can write

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \quad \text{which makes } |\vec{S}| = I$$

Power thru surface for EM wave will be

$$P = \int \underbrace{\vec{S} \cdot d\vec{A}}_{\text{only } \vec{S} \perp \text{ surface } \vec{A} \text{ matters}}$$

area of surface

so $\vec{S} = S\hat{z}$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{E_0 \cos(kz - wt) B_0 \cos(kz - wt)}{\mu_0} \hat{x} \times \hat{y}$$

$$= \frac{E_0 B_0}{\mu_0} \underbrace{\cos^2(kz - wt)}_{} \hat{z}$$

$S(z,t)$ is always > 0 since it involves the flow of energy

Intensity you measure is averaged over many cycles for sinusoidal wave:

$$\bar{I} = \frac{1}{T}\int_0^T \frac{\vec{E}\times\vec{B}}{\mu_0}\,dt = \frac{1}{T}\frac{E_0B_0}{\mu_0}\underbrace{\int_0^T \cos^2(kz-\omega t)\,dt}_{T/2}$$

so $\bar{I} = \frac{E_0B_0}{2\mu_0} = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{B_0^2 c}{2\mu_0}$

ex: laser has $E_0 = 1000\,V/m$

intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\cdot 8.85\times10^{-12} \times 1000^2$

$= 4.43\times10^{-6}\,W/m^2$

ex: laser pointer has power $P = 5\,mW$ w/ diameter 2mm

Area $A = \pi\left(\frac{1}{2}d\right)^2 = \pi\times\left(10^{-3}m\right)^2 = \pi\times10^{-6}\,m^2$

Intensity $I = P/\text{area} = 5\times10^{-3}\,W / \pi\times10^{-6}\,m^2$

$= 1592\,W/m^2$

$I = \frac{1}{2}\varepsilon_0 E_0^2 = 1592$

$E_0 = \sqrt{\frac{2\times1592}{8.85\times10^{-12}}} = 18.96\times10^{6}\,V/m$

this a very large electric field

but it only occurs over a short distance

Average Intensity of light hitting earth from sun

$I \simeq 1400\ W/m^2$ at top of atmosphere

atmosphere is absorbing, maybe lets 70% thru

so $I_m \sim 1000\ W/m^2$ at surface

$\Rightarrow$ and will vary w/ latitude & time of day

so estimate average intensity over a day is

$\sim \frac{1}{2} I_{max}$ for 12 hours, then drops to ϕ at nite

so during day, $\bar{I} = 500\ W/m^2$

$\Rightarrow$ average home uses $\sim 1\ kW$ averaged over a day
want to install solar panels to run the house during day & also charge batteries for night

so need $\bar{P} = 2\ kW$ during 12 hours daytime to use half for powering home, half for charging batteries for night-time

$\Rightarrow$ solar panels efficiency $\sim 20\%$ at best

so power saved $\bar{P}_s = 0.2 \times \bar{P}_{in}$ and we want

$\bar{P}_s$ to be $2\ kW$

so $\overline{P}_{in} = \frac{\overline{P}_s}{0.2} = \frac{2\,kW}{0.2} = 10\,kW$ into panels

this power comes from $\overline{I}$ hitting panels

$\overline{I} = \frac{\overline{P}}{\text{Area}}$ so Area $A = \overline{P}/\overline{I}$

$$A = \frac{\overline{P}}{\overline{I}} = \frac{10\,kW}{\frac{1}{2}W/m^2} = 20\,m^2$$

$$= 20\,m^2 \times \left(\frac{3.28\,ft}{m}\right)^2 = 215\,ft^2$$

$= 15\,ft \times 15\,ft$ of panels

<u>with</u> batteries!

Residential solar panels: 66×40 in

$$= 5\tfrac{1}{2}\,ft \times 3\tfrac{1}{3}\,ft$$

$$= \frac{11}{2} \times \frac{10}{3} = \frac{110}{6} = 18\,ft^2$$

so need ~ 11 panels

This is a ower estimate

- shade, power use fluctuations, cloudy or rainy skies,
- In summer load can go up x2-3 or more

AC in summer - load can go upto

Radiation pressure

Intensity $I = \frac{\text{power}}{\text{area}} = \frac{\text{energy}}{\text{sec} \cdot \text{area}}$

units are $\frac{J}{s\,m^2}$ and $1\,J = 1\,N\text{-}m$

so units of I: $\frac{N\,m}{s\,m^2} = \underbrace{\frac{N}{m^2}}_{\text{pressure}} \cdot \frac{m}{s}$ ← velocity

pressure P is called "radiation pressure"

⇒ this pressure = force/area and force is rate of change of momentum

→ light carries momentum!

light on a body can be absorbed or reflected

absorbed

initial light momentum = final body momentum which comes from radiation pressure

$$I = P \cdot c \Rightarrow P = I/c$$

ex: pressure from sunlight at top of atmosphere $I = 1400\,W/m^2$

$$P = \frac{1400\,W/m^2}{3\times10^8\,m/s} = 4.7\times10^{-6}\ \underbrace{N/m^2}_{Pa}$$

pascal

if we have "light sail" w/ area A, force will be:

$$P = F/A \Rightarrow F = PA$$

for F = 1 N (2.2 lbs) then can solve for A

$$A = F/P = \frac{1\,N}{4.7\times10^{-6}\,N/m^2} = 2.1\times10^5\,m^2$$

$$= (463\,m)^2 \approx (1500\,ft)^2 \sim 0.3\,mi$$

on a side!

reflected light gives x2 more momentum due to momentum conservation, so x2 pressure

momentum conservation:

	light	body
initial	$p_{in} \rightarrow$	0
final	$\leftarrow p_{out}$	$p \rightarrow$

so $p_{in} = -p_{out}$ for total reflection

so $p_{in} = p_{out} + p = -p_{in} + p$

$$\therefore\ p = 2p_{in}$$

so make sail reflective, $A = 1.0 \times 10^5\, m^2$

$\sim (325\,m)^2 \sim (1000\,ft)^2$